

Quark Mass and the QCD Transition

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Abstract. We discuss different aspects of the quark mass dependence of the chiral and deconfinement transitions. We make predictions for locating the deconfining critical point, by analyzing the m_q dependence of the ratio of imaginary and real mass of the Polyakov loop. We then present an effective theory that allows the extraction of coupling constants from available lattice data for the temperature dependence of the chiral and Polyakov loop condensates at different quark masses.

Theoretical developments accompanied by recent numerical results from the lattice have advanced our understanding of QCD. There are still numerous important questions however that need to be answered. For instance: Where does the QCD point lie in the (m_s, m_q) and in the (T, m_q) phase diagrams? Which mechanism drives the transition for a given quark mass? Is the QCD transition more deconfining or chiral symmetry restoring? See [1] for related discussion. As a step towards answering these questions, first, we discuss the location of the deconfining critical point in the (T, m_q) plane. Then, we introduce a general theory for the Polyakov loop and chiral field at finite quark mass, and discuss its fit to lattice data.

The pure gauge sector of QCD with N colors, i.e. $m_q = \infty$, has global Z_N symmetry associated with the center of the $SU(N)$ gauge group. The Polyakov loop ℓ , charged under Z_N , serves as order parameter for the deconfining transition. ℓ is real for $N = 2$ and the Z_2 symmetry breaking deconfinement transition is of second order. For $N = 3$ the Polyakov loop is complex and transforms as $\ell \rightarrow \exp(2\pi i/3)\ell$. Accordingly, the expectation value $\langle \ell \rangle = 0$ at low temperatures, when Z_3 is unbroken, and $\langle \ell \rangle \neq 0$ above the deconfinement critical temperature T_d . The deconfinement transition for $N = 3$ is weakly first order. This has been verified by lattice QCD [1].

In [2] an effective theory for the Polyakov loops, the Polyakov Loop Model (PLM) has been constructed[‡]. The potential for $N = 3$, up to quartic terms, is

$$V(\ell) = -b_1 \frac{\ell + \ell^*}{2} - \frac{b_2}{2} |\ell|^2 - \frac{b_3}{3} \frac{\ell^3 + \ell^{*3}}{2} + \frac{1}{4} (|\ell|^2)^2. \quad (1)$$

In a mean field analysis all coupling constants, except $b_2(T)$, are temperature independent. There are two masses, associated with the real $\ell_r = \text{Re } \ell$ and imaginary $\ell_i = \text{Im } \ell$ parts of the Polyakov loop. These are defined from the two-point functions

$$\langle \ell_r(x) \ell_r(0) \rangle - \ell_0^2 \sim \frac{\exp(-m_r x)}{x}, \quad \langle \ell_i(x) \ell_i(0) \rangle \sim \frac{\exp(-m_i x)}{x}, \quad (2)$$

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[‡] Some other approaches to study deconfinement are in [3].

and thus characterize the exponential tail of the correlators. $\langle \ell \rangle = \ell_0 > 0$ is real. Denote the two minima $\ell^- = \ell(T_d^-)$ and $\ell^+ = \ell(T_d^+)$. The masses at T_d^+ are then

$$m_r^2 = -b_2 - 2b_3\ell^+ + 3(\ell^+)^2, \quad m_i^2 = -b_2 - 2b_3\ell^+ + (\ell^+)^2. \quad (3)$$

For $b_1 = 0$ the potential is Z_3 symmetric. The solutions are

$$\ell_0^- = 0, \quad \ell_0^+ = \frac{2}{3}b_3, \quad b_2 = -\frac{2}{9}b_3^2, \quad (4)$$

and the ratio of the masses at T_d^+ is $m_i/m_r = 3$ [4]. This value is twice the one expected from lowest order perturbative calculations, valid at high temperatures. The PLM thus predicts an increase of m_i/m_r as $T \rightarrow T_d^+$, prediction supported by the lattice [5]. In principle, m_i/m_r can receive contributions from pentic and hexatic terms (see discussion in [4]).

What happens to m_i/m_r when dynamical quarks are added in the theory? How does this ratio change for finite quark masses? For any finite m_q the Z_3 symmetry is explicitly broken, and the linear term b_1 is turned on in (1). This corresponds to tilting the potential and shifting the degenerate minima at T_d towards each other, and thus the transition becomes weaker first order. An increasing symmetry breaking is equivalent to decreasing the quark masses from infinity. An explicit functional relation for $b_1(m_q)$ fitted for lattice data, together with the m_q dependence of T_d is discussed in [6]. For some value of m_q there is only one minima and the transition is of second order. This is the deconfining critical point D, shown in the left panel of Fig. 1. When further decreasing the quark mass the phase transition becomes a crossover. To determine the influence of quark mass we derived a set of differential equations

$$\frac{\partial \ell_0^-}{\partial b_1} = \frac{1}{m_-^2} \left(1 + \frac{\partial b_2}{\partial b_1} \ell_0^- \right), \quad \frac{\partial \ell_0^+}{\partial b_1} = -\frac{1}{m_+^2} \left(1 + \frac{\partial b_2}{\partial b_1} \ell_0^+ \right), \quad \frac{\partial b_2}{\partial b_1} = -\frac{2}{\ell_0^- + \ell_0^+} \quad (5)$$

and solve them together with the initial conditions (4) numerically. Here $V''(\ell^-) = m_-^2$ and $V''(\ell^+) = m_+^2$, and $b_3 = 0.9$. Note that the third term is always negative, leading to decreasing T_d with m_q (see the slope of the upper 1st order critical line in the left panel of Fig. 1). The mass ratio at the critical temperature in terms of the symmetry breaking is shown in the right panel of Fig. 1. We find that as the quark mass decreases from infinity the ratio slightly increases. The second order deconfining critical point D is reached for $b_1 = 0.027$ and here the mass ratio becomes divergent. This means that only the real part ℓ_r , not the ℓ_i , becomes massless at D, and thus Z_3 is a symmetry group here. Following the behavior of m_i/m_r on the lattice would allow for locating D, and also to determine the quark mass that this point corresponds to.

Another well studied sector of QCD is in the limit of zero and small quark masses. In this regime Z_N is always broken, but chiral symmetry is restored above a critical temperature T_c . The order parameter is the chiral condensate σ . The transition is second order for two, and first order for three flavors of massless quarks. For finite quark masses chiral symmetry is explicitly broken. Increasing m_q from zero means weakening the first order phase transition. For some m_q the first order critical line turns into a second order chiral critical point, C in the left panel of Fig. 1. For even larger m_q the transition is a crossover. Effective field theories have been used to analyze this sector [8].

Deconfinement and chiral symmetry restoration are two phenomena very different in nature. For realistic quark masses both the chiral and Z_N symmetries are explicitly broken, and likely that both transitions are in the crossover domain [7]. Here neither σ , nor ℓ can play the role of a true order parameter. However, lattice simulations

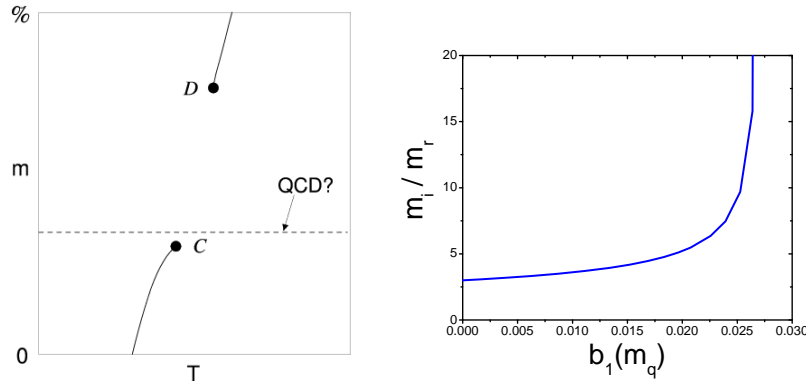


Figure 1. Left panel: Temperature–quark mass phase diagram. From [7]. Right panel: Quark mass dependence of the ratio of imaginary and real mass of the Polyakov loop at the corresponding critical temperature.

show that the susceptibilities associated with these two quantities peak at the same temperature, when quarks are in the fundamental representation of the gauge group [9], indicating that the transitions occur at the same critical temperature. Similar results were obtained also in terms of the chemical potential [10].

In [11] we provided an explanation within a generalized Ginzburg-Landau theory to why for $m_q = 0$ chiral symmetry restoration leads to deconfinement. The analysis was based on the following general idea: The behavior of an order parameter induces a change in the behavior of non-order parameters at the transition [12], via the presence of a possible coupling between the fields, $g_1 \ell \sigma^2$. In [11] we assumed $g_1 > 0$, which is confirmed by extracting it from the lattice [13].

Here we extend the analysis of [11] for finite quark masses. We can then study not only the qualitative relation between deconfinement and chiral symmetry restoration in terms of quark masses, but also can provide quantitative statements after extracting the coupling constants from lattice data [13]. Recent lattice results for the temperature dependence of the chiral condensate for three degenerate massive quarks were reported in [14]. The first results for the behavior of the renormalized Polyakov loop [15, 16] as a function of temperature, under the same lattice conditions, were reported in [16].

We now write down the most general renormalizable effective Lagrangian that can be built from the chiral field σ and the Polyakov loop ℓ . The contribution to the potential $V = V_{Pl} + V_{ch} + V_{int}$ from the Polyakov loop is

$$V_{Pl}(\ell) = g_0 \ell - \frac{b_2}{2} \ell^2 - \frac{b_3}{3} \ell^3 + \frac{1}{4} \ell^4. \quad (6)$$

Here $g_0 = -b_1$ §. For the chiral potential we adopt the linear σ model, discarding all fields but the σ , since in relation to our discussion these will not play a role [13]:

$$V_{ch}(\sigma) = \frac{m^2}{2} \sigma^2 + \frac{\lambda}{4} \sigma^4 - H \sigma. \quad (7)$$

Here $H = f_\pi m_\pi^2$, with m_π and f_π the vacuum pion mass and decay constant. The possible interactions terms are

$$V_{int}(\ell, \sigma) = g_1 \ell \sigma^2 + g_2 \ell^2 \sigma^2 + \bar{g}_1 \ell^2 \sigma + \bar{g}_2 \ell \sigma. \quad (8)$$

§ This potential is written only for the real part of the Polyakov loop, since we are interested in the behavior of its expectation value, which can always be chosen real at $\mu = 0$.

In a mean field analysis the field expectation values to first order in m_q are

$$\langle \ell \rangle \simeq -\frac{g_0}{m_\ell^2} - \frac{g_1}{m_\ell^2} \langle \sigma \rangle^2 - \frac{\bar{g}_2}{m_\ell^2} H \langle \sigma \rangle \quad (9)$$

and $\langle \sigma \rangle$ is the solution of

$$\langle \sigma \rangle^3 + \frac{g_1 \bar{g}_2}{2\lambda m_\ell^2} H \langle \sigma \rangle^2 - \frac{m_\sigma^2}{2\lambda} \langle \sigma \rangle + \frac{H}{2\lambda} \left(1 + \frac{g_0 \bar{g}_2}{m_\ell^2} \right) \simeq 0. \quad (10)$$

For $H = 0$ we recover the results of [11], as expected. For light quarks of three degenerate flavors, the Polyakov loop mass is identified as the inverse of the screening radius and is extracted from the lattice [16]. For the σ mass we assume $m_\sigma^2(T) = a(bT/T_c - 1)$ as is customary in a Ginzburg-Landau theory. Then using (9) and (10) we fit the data from [16] and [14]. The couplings extracted this way confirm the assumption made in [11] in that $g_1 > 0$ and $g_0 < 0$. Accordingly, for massless quarks ($H = 0$) the decrease of the order parameter, the chiral condensate induces an increase in the Polyakov loop, as one can see directly in (9). Which field drives the transition in the case of massive quarks can be inferred from the knowledge of the couplings. These results will be presented elsewhere [13].

With our effective model, we will be able to make predictions relevant for the phenomenology of heavy ion collisions. The extension of this analysis to the chemical potential axis of the phase diagram is relevant not only in the context of the critical point in the (T, μ) plane, but also in the understanding of the phase structure of QCD.

Acknowledgements

Special thanks to my collaborators A.Dumitru, E.S.Fraga, F.Sannino, K.Tuominen. I thank P.Petreczky and R.Pisarski for discussions and A.Dumitru for carefully reading the manuscript. I am grateful to the Theoretical Physics Institute at J.W. Goethe University in Frankfurt for the kind hospitality, where I am a Humboldt Fellow.

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